LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION - **PHYSICS**

FIRST SEMESTER – **APRIL 2013**

PH 1820 - MATHEMATICAL PHYSICS - I

Date : 06/05/2013 Dept. No. Max. : 100 Marks
Time : 9:00 - 12:00

PART A

Answer **ALL** questions 10 x 2 = 20

1) Distinguish between the Gauss elimination method of solving the system of linear equations with and without pivoting.

2) Write down the algorithm for the Regula Falsi method.

3) Determine and graph the loci represented by $0 < \text{Im } 1/z < 1$.

4) What do you understand by “the residue of a complex function f (z)”?

5) State the Cauchy- Schwarz inequality of the scalar product of two vectors.

6) Distinguish between proper and improper orthogonal transformations.

7) Give an illustration of occurrence of tensor in physics.

8) Define the contravariant and covariant vectors of rank one by their transformation properties.

9) State the orthonormality property of the Legendre polynomials.

10) What are spherical harmonics?

PART B

Answer any **FOUR** questions 4 x 7.5 = 30

11) Find $\sqrt[3]{18}$ correct to two decimal places by Newton – Raphson method assuming 2.5 as the initial approximation.

12) Determine whether the function $v = e^x \sin y$ is harmonic. If your answer is yes, find a corresponding analytic function $f(z) = u(x, y) + i\, v(x, y)$.

13) (a) Prove that the eigenvalues of a Hermitian matrix are real and any two eigenvectors belonging to distinct eigenvalues are mutually orthogonal to each other.(b) Prove that the eigenvalues of an anti-

Hermitian matrix are either zero or pure imaginary and any two eigenvectors belonging to distinct eigenvalues are orthogonal to each other. (4 + 3 ½)

14) Obtain the metric for a 3-dimensional space in terms of (i) polar coordinates and (ii) Cartesian coordinates (4 + 3 ½)

15) Prove that the gamma function Γ(x+1) = x Γ(x) and if x is a non-negative integer Γ(x+1) = x! (4+3½)

PART C

Answer any **FOUR** questions 4 x 12.5 = 50

16) Solve by Gauss elimination method (both without pivoting and with pivoting) the system of linear equations $6x - y - z = 19$; $3x + 4y + z = 26$; $x + 2y + 6z = 22$.

17) (a) Using the contour integration, evaluate the following real integral. $\int_0^{\infty} \frac{dx}{(1+x^2)^3}$ (b) Evaluate the following integral using Cauchy's integral formula $\int_c \frac{4-3z}{z(z-1)(z-2)}$ dz, where C is the circle $|Z| = 3/2$. (6 ½ + 6)

18) (a) Explain the Schmidt orthogonalisation procedure. (b) Construct an orthonormal set of three linearly independent vectors from the given set of vectors $a = \begin{pmatrix}1\\1\\0\end{pmatrix}$; $b = \begin{pmatrix}2\\0\\1\end{pmatrix}$; $c = \begin{pmatrix}1\\1\\1\end{pmatrix}$. (6 ½ + 6)

19) What is a fully antisymmetric tensor? Express (a) the vector product of two vectors (b) the commutation relation between the components of angular momentum in quantum mechanics by using the fully antisymmetric tensor of rank three. (2 ½ + 5 + 5)

20) (a) Solve the Legendre differential equation $(1 - x^2)\frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + n\,(n+1)\,y = 0$ by the power series method. (b) Establish the orthonormality relation $\int_{-1}^{+1} P_n(x)P_m(x)dx = \begin{cases} 0 \; if \; n \neq m \\ \frac{2}{2n+1} \; if \; n = m \end{cases}$ where $P_n(x)$ is the Legendre polynomial of order n. (6 ½ + 6)

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